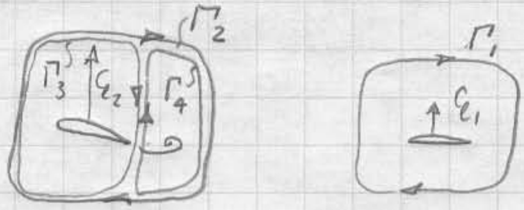
UE Fluids Problem 1 Solution S'08

1) By Kelvin's Theorem, $\Gamma_2 = \Gamma_1$



In general, $L' = \rho V_\infty \Gamma = \frac{1}{2}\rho V_\infty^2 c\, c_\ell$

or $\Gamma = \frac{1}{2} c V_\infty c_\ell$ (for airfoil)

$\therefore \Gamma_1 = \frac{1}{2} c V_\infty c_{\ell_1} = \frac{1}{2}\cdot 1\text{m}\cdot 30\text{m/s}\cdot 0.5 = 7.5\ \text{m}^2/\text{s} = \Gamma_2$

But we also know $\Gamma_3 + \Gamma_4 = \Gamma_2$ by geometry

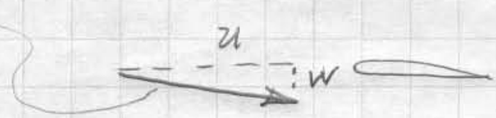
And since $\Gamma_3 = \frac{1}{2} c V_\infty c_{\ell_2} = \frac{1}{2}\, 1\text{m}\cdot 30\text{m/s}\cdot 1.5 = 22.5\ \text{m}^2/\text{s}$

$\therefore \boxed{\Gamma_{vortex} = \Gamma_4 = \Gamma_2 - \Gamma_3 = -15\ \text{m}^2/\text{s}}$ (counterclockwise)

2a) Observer sees: • horizontal $\boxed{u = V_\infty = 30\text{m/s}}$

• vertical $\boxed{w = \frac{\Gamma_{vortex}}{2\pi d} = \frac{-15\ \text{m}^2/\text{s}}{2\pi\cdot 10\text{m}} = -0.2387\ \text{m/s}}$

$\vec{V}_{apparent} = u\hat{\imath} + w\hat{k}$



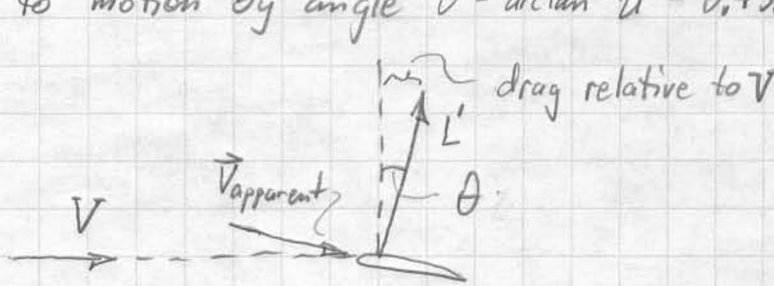
2b) Airfoil thinks it's in a freestream $\vec{V}_{apparent}$.

Apparent Lift force is $\perp$ to $\vec{V}_{apparent}$: $L' = \frac{1}{2}\rho|\vec{V}_{apparent}|^2 c\, c_\ell \simeq \frac{1}{2}\rho V^2 c\, c_\ell$

Apparent Drag force is $\parallel$ to $\vec{V}_{apparent} = 0$ (d'Alembert)

This Lift force is tilted back relative to motion by angle $\theta = \arctan\frac{-w}{u} = 0.456^\circ$

True drag relative to V is



$D' = L' \sin\theta$

or $\boxed{c_d = c_\ell \sin\theta = 0.0119}$